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# Mark Scheme (Results)

## Summer 2014

## Pearson Edexcel GCE in Statistics S4R (6686/01R)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- $\overline{\phantom{x}}$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

| Question | Scheme | Marks |
|---|---|---|
| **1. (a)** | [New – standard = ] $d$ : 7, 4, –5 , 18, –12, 18, 11, 13. | M1 |
| | $\bar{d} = 6.75$ | M1 |
| | $s_d{}^2 = \dfrac{1172 - 8\times 6.75^2}{7} = 115.3571...$ or $s_d = 10.7404...$ | M1 |
| | $H_0 : \mu_d = 0$ $\quad H_1 : \mu_d > 0$ | B1 |
| | $t_7 = \dfrac{6.75}{\frac{s_d}{\sqrt{8}}} = 1.7775...$ or $\dfrac{c}{s_d/\sqrt{8}} = 1.895 \therefore$ CR $c >$ awrt 7.2 | M1 A1 |
| | awrt **1.78** | |
| | $t_7(5\%)$ one tail critical value is **1.895** (or prob. = 0.05935…) | B1 |
| | Not significant. | |
| | There is insufficient evidence that the new medicine is better or the new medicine is not recommended. | A1ft (8) |
| **(b)** | Need the differences between levels triggering coughing to be normally distributed | B1 (1) |
| | | **(9 marks)** |
| | **Notes** | |
| **(a)** | 1st M1 for attempting the $d$s | |
| | 2nd M1 for attempting $\bar{d}$ | |
| | 3rd M1 for attempting $s_d$ or $s_d{}^2$ | |
| | 1st B1 for both hypotheses correct in terms of $\mu$ or $\mu_d$ | |
| | 4th M1 for attempting the correct test statistic $\dfrac{6.75}{s_d/\sqrt{8}}$ or $p$ = awrt 0.06 or $\dfrac{c}{10.7/\sqrt{8}} = t$ value | |
| | 1st A1 1.78 or awrt 0.06 or awrt 7.2 | |
| | 2nd B1 1.895 or awrt 0.06 | |
| | 2nd A1ft for a correct comment in context based on their test statistic and their cv. | |
| **(b)** | B1 for a comment that mentions “differences” and “normal” distribution | |

| Question | Scheme | Marks |
|---|---|---|
| **2. (a)** | [$X$ = no. of defects in 4 square metres.] $X$ ~Po(6) | |
| | [Size =] P($X$ > 10) + P($X$ = 9 or 10)P($X$ > 10) | M1 |
| | = ( 1 – 0.9574) + (0.9574 – 0.8472)(1 – 0.9574) | M1A1 |
| | = 0.04729… = awrt **<u>0.0473</u>** | A1 (4) |
| **(b)** | $Y$ ~ Po(8) | B1 |
| | Power = $1 - (\mathrm{P}(X \leq 8) + [\mathrm{P}(X = 9) + \mathrm{P}(X = 10] \times \mathrm{P}(X \leq 10))$ | M1 |
| | Or $(1 - \mathrm{P}(X \leq 10)) + [\mathrm{P}(X = 9) + \mathrm{P}(X = 10] \times (1 - \mathrm{P}(X \leq 10))$ | |
| | = (1 – 0.8159) + (0.8159 – 0.5925)(1 – 0.8159) | |
| | = 0.22522… = awrt **<u>0.225</u>** | A1 (3) |
| | | **(7 marks)** |

| | Notes |
|---|---|
| **(a)** | 1st M1 for a correct expression/selection of probabilities |
| | 2nd M1 for use of Po(6) and at least one correct prob. seen |
| | May see $\mathrm{P}(X = 9) = \frac{\mathrm{e}^{-6}6^9}{9!}$ =0.06883... or $\mathrm{P}(X = 10) = \frac{\mathrm{e}^{-6}6^{10}}{10!} = 0.04130...$ |
| | 1st A1 for a fully correct expression |
| | 2nd A1 for awrt 0.0473 |
| **(b)** | B1 for evidence of <u>use</u> of Po(8) |
| | M1 for an expression of the correct form with at least one correct prob. |
| | A1 for awrt 0.225 |

| Question | Scheme | Marks |
|---|---|---|
| **3. (a)** | $H_0 : \sigma_A^{\,2} = \sigma_B^{\,2}$ $\quad H_1 : \sigma_A^{\,2} \neq \sigma_B^{\,2}$ | B1 |
| | $\left(F_{8,11} =\right)\dfrac{2.98^2}{2.33^2} = (1.6357...)$ | M1 |
| | $F_{8,11}$ 10% (two-tail) cv = **2.95** (or prob. = awrt 0.22) | B1 |
| | Not significant so can accept the assumption that variances are equal. | A1 |
| | | (4) |
| **(b)** | $H_0 : \mu_A = \mu_B$ $\quad H_1 : \mu_A \neq \mu_B$ | B1 |
| | $s_p^{\,2} = \dfrac{8\times 2.98^2 + 11\times 2.33^2}{19}, = 6.88216...$ or $s_p = 2.62338...$ | M1, A1 |
| | $\left(t_{19} =\right)(\pm)\dfrac{7.13-6.23}{s_p\sqrt{\dfrac{1}{9}+\dfrac{1}{12}}} = (\pm)0.7780047...$ = awrt **<u>0.778</u>** | M1<br>A1 |
| | $t_{19}$ (0.05) two-tail cv = **2.093** | B1 |
| | [Not significant]<br>Insufficient evidence of a **<u>difference in mean</u>** milk **<u>yields</u>** between the two **<u>breeds</u>** | A1 |
| | | (7) |
| **(c)** | Test in part(b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal. | B1 |
| | | (1) |
| | | **(12 marks)** |

| | Notes |
|---|---|
| | |
| **(a)** | 1st B1allow $\sigma$ or $\sigma^2$<br>M1 for use of the correct test statistic |
| **(b)** | 1st M1 for attempting $s_p$ or $s_p^{\,2}$<br>1st A1 for awrt 6.90 or 2.63<br>2nd M1 for use of a correct test statistic<br>2nd A1 for awrt 0.77 (accept $\pm$)<br>2nd B1 for 2.093 (allow $\pm$ 1.729 for one-tailed $H_1$) |

| Question | Scheme | Marks |
|---|---|---|
| 4. (a) | $s^2 = \dfrac{42397 - 10\times\left(\dfrac{619}{10}\right)^2}{9} = 453.433...$ = awrt <u>453</u> | B1 |
| | $H_0 : \sigma = 19.71$ (or $\sigma^2 = ...$) $H_1 : \sigma > 19.71$ (or $\sigma^2 > ...$) | B1 |
| | $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_9$ test statistic = 10.5046… = awrt **<u>10.5</u>** | M1A1 |
| | $\chi^2_9$ (0.05) cv = **16.919** | B1 |
| | Not significant so insufficient evidence that the **<u>scores</u>** of the **<u>students</u>** are more varied than normal.<br><u>Or</u><br>**<u>Admission tutor's</u>** claim is not supported | A1<br>(6) |
| **(b)** | $\chi^2_{29}(0.01)$ cv = **49.588** | B1 |
| | Reject $H_0$ if $\dfrac{29S^2}{19.71^2} > 49.588$ | M1 |
| | So critical region is $S^2 > 664.281...$ = awrt **<u>664.281</u>** | A1cso<br>(3) |
| **(c)** | P(Type II error) = P( $S^2 < 664.281... \mid \sigma = 22.20$) <u>or</u> $P\left(\chi^2_{29} < \dfrac{664.281...\times 29}{22.20^2}\right)$ | M1<br>A1ft |
| | = P( $\chi^2_{29} < 39.088…$) = 0.90 = awrt **<u>0.90</u>** | A1<br>(3) |
| | | **(12 marks)** |

| | Notes |
|---|---|
| **(a)** | M1 for use of the correct test statistic |
| **(b)** | M1 for use of a correct expression (LHS) only |
| **(c)** | M1 for a correct probability expression involving $S^2$ or $\chi^2_{29}$ . Ft their CR, may be implied by a correct answer<br>1st A1ft for a correct probability expression with $\chi^2_{29}$ but ft their CR, may be implied by a correct answer |

| Question | Scheme | Marks |
|---|---|---|
| **5. (a)(i)** | $\bar{x} = \left(\frac{880}{15} =\right)\ 58.\dot{6}$ or awrt 58.7 | B1 |
| | $s_x^{\,2} = \left(\frac{54892 - 15\times 58.\dot{6}^2}{14} =\right) 233.238...$ | B1 |
| | $t_{14}(0.025)$ cv = 2.145 | B1 |
| | 95% CI for $\mu$ is $58.\dot{6} \pm 2.145 \times \sqrt{\frac{233.238...}{15}}$ | M1 |
| | = (50.209…, 67.124…) = awrt **(50.2, 67.1)** | A1, A1 |
| **(ii)** | $\chi_{14}^{\,2}(0.025) = 5.629$, $\chi_{14}^{\,2}(0.975) = 26.119$ | B1, B1 |
| | 95% CI for $\sigma^2$ is given by: $5.629 < \frac{14 s_x^{\,2}}{\sigma^2} < 26.119$ | M1 |
| | = (125.017… , 580.0911…) | A1 |
| | So 95% CI for $\sigma$ is = (11.1811… , 24.0850…) = awrt **(11.2, 24.1)** | A1 |
| | | (11) |
| **(b)** | Require P($S > d$) $\leq 0.80$ i.e. $P\left(Z > \frac{d-\mu}{\sigma}\right) \leq 0.80$ | |
| | From tables $\pm$ 0.8416 | B1 |
| | So require: $\frac{d-\mu}{\sigma} > -0.8416$ | M1 |
| | i.e. $d > \mu - 0.8416\sigma$ | A1 |
| | Worst case is when $\mu = \mu_{\max}$ and $\sigma = \sigma_{\min}$ | M1 |
| | So $d > 67.1 - 0.8416 \times 11.2$ (= 57.674…) so they should set a **pass mark of 58** | A1 (5) |
| | | **(16 marks)** |

| | Notes |
|---|---|
| **(a)** | 1st M1 ‘ their $\bar{x}$ ’ $\pm$ *t value* $\times \frac{\text{'their } s\text{'}}{\sqrt{15}}$ |
| | 1st A1 for awrt 50.2 |
| | 2nd A1 for awrt 67.1 |
| | 2nd M1 for use of their values in $\chi^2 < \frac{14 s^2}{\sigma^2} < \chi^2$ |
| | 3rd A1 for awrt 125 or 580 |
| | 4th A1 for awrt 11.2 and 24.1 |
| **(b)** | 1st M1 for forming a correct expression in $d$, $\mu$, $\sigma$ and their $z$ value |
| | 2nd M1 for using their top value from CI for $\mu$ and lowest value for CI for $\sigma$ |

| Question | Scheme | Marks |
|---|---|---|
| **6. (a)** | $\mathrm{E}(X)=\int_0^a x\,\frac{2}{a^2}x\,\mathrm{d}x=\left[\frac{2}{a^2}\frac{x^3}{3}\right]_0^a=\underline{\frac{2a}{3}}$ | B1cso |
| | $\mathrm{E}(X^2)=\int_0^a x^2\,\frac{2}{a^2}x\,\mathrm{d}x=\left[\frac{2}{a^2}\frac{x^4}{4}\right]_0^a=\frac{a^2}{2}$ so $\sigma^2=\frac{a^2}{2}-\frac{4a^2}{9}=\underline{\frac{a^2}{18}}$ | M1<br>A1 |
| | So $\mathrm{E}(\bar{X})=\mu=\frac{2a}{3}$ and $\mathrm{Var}(\bar{X})=\frac{\sigma^2}{n}=\frac{a^2}{18n}$ | A1cso (4) |
| **(b)** | $p=\underline{\frac{3}{2}}$ and $\mathrm{Var}(S)=\frac{9}{4}\mathrm{Var}(\bar{X})=\underline{\frac{a^2}{8n}}$ | B1, B1ft (2) |
| **(c)** | $\mathrm{E}(M)\to a$ as $n\to\infty$, and $\mathrm{Var}(M)\to 0$ as $n\to\infty$<br>So $M$ is a consistent estimator of $a$ | B1, B1<br>dB1 (3) |
| **(d)** | $q=\underline{\frac{2n+1}{2n}}$, $\mathrm{Var}(T)=\frac{\cancel{(2n+1)^2}}{4n^{\cancel{2}}}\times\frac{\cancel{n}}{(n+1)\cancel{(2n+1)^2}}a^2,=\underline{\frac{a^2}{4n(n+1)}}$ | B1,<br>M1,<br>A1 (3) |
| **(e)** | $\frac{a^2}{4n(n+1)}<\frac{a^2}{8n}\Leftrightarrow 2<n+1\Leftrightarrow 1<n$ So $\mathrm{Var}(T)<\mathrm{Var}(S)$ | M1<br>A1 |
| | So (since both are unbiased) choose $T$ since it has the lower variance | A1cso. (3) |
| **(f)** | $m=7.8$ so using $t$ gives estimate of $\frac{11}{10}\times 7.8,=8.58$ [NB $\bar{x}=6$ and $s$ gives 9] | M1,<br>A1ft (2) |
| **(g)** | Using $\mathrm{Var}(T)=\frac{a^2}{120}$; so standard error is $\frac{8.58}{\sqrt{120}}$, = awrt **<u>0.78</u>** [NB $s$ gives $\frac{a}{\sqrt{40}}=1.42$] | M1;A1 (2)<br>**(19 marks)** |
| | **Notes** | |
| **(a)** | 1st B1 for some working to establish $\mu$. Allow median of triangle for example.<br>1st M1 for correct method for $\sigma^2$ | |
| **(b)** | 2nd B1ft ft their value of $p$ | |
| **(c)** | 3rd dB1 dependent on both of first 2 Bs in (c) for concluding that $M$ is consistent | |
| **(d)** | M1 for correct use of $\mathrm{Var}(T)=q^2\mathrm{Var}(M)$ for their $q$. | |
| **(e)** | M1 for attempt to compare Var($T$) and Var($S$)<br>1st A1 for clearly establishing that Var($T$) < Var($S$)<br>2nd A1 for choosing $T$ and stating variance is smaller<br>SC M0 A0 B1 for T because it has a smaller variance | |
| **(f)** | M1 for using their estimator chosen in (e) | |
| **(g)** | M1 for using their Variance formula to calculate std. error. subst in $n$=4 and their (f) | |

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